Driven Dynamics and Rotary Echo of a Qubit Tunably Coupled to a Harmonic Oscillator

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We have investigated the driven dynamics of a superconducting flux qubit that is tunably coupled to a microwave resonator. We find that the qubit experiences an oscillating field mediated by off-resonant driving of the resonator, leading to strong modifications of the qubit Rabi frequency. This opens an additional noise channel, and we find that low-frequency noise in the coupling parameter causes a reduction of the coherence time during driven evolution. The noise can be mitigated with the rotary-echo pulse sequence, which, for driven systems, is analogous to the Hahn-echo sequence.

Circuit quantum electrodynamics implemented with superconducting artificial atoms and microwave resonators has emerged as a framework for studying on-chip light-matter interactions [1–3]. It has enabled a range of experiments including lasing [4], the creation [5–7] and detection [8] of arbitrary Fock states, and microwave photon-correlation measurements [9,10]. Microwave resonators also provide a means to couple distant qubits [11,12] and, in this role, have been used to implement quantum algorithms in superconducting circuits [13] and to develop quantum computer architectures [14]. However, the coupling of a qubit to a resonator also influences the qubit coherence, for example, by modifying its relaxation rate through the Purcell effect [15]. Such adverse effects can be avoided by making the qubit-resonator coupling tunable [16–19], which also provides advantages when coupling multiple qubits through a single resonator, since it allows the effective qubit-qubit interactions to be turned on and off.

In this work, we study the driven dynamics and the dephasing of a flux qubit [20] that is tunably coupled to a lumped-element harmonic oscillator [21]. We find that the resonator mediates an indirect driving field that interferes with the direct drive set by the qubit-antenna coupling, thereby modifying both the amplitude and the phase of the net driving-field. The tunable coupling allows the indirect driving to be switched off, but it also opens an additional channel for noise to couple into the system. Fluctuations in the coupling parameter translate into effective driving-field amplitude noise, which reduces the qubit coherence during driven evolution. We show that the qubit dephasing due to amplitude noise (whether due to tunable coupling or

otherwise) can be mitigated by a rotary echo [22], a pulse sequence originally developed for nuclear magnetic resonance.

The device, shown in Fig. 1(a), consists of a flux qubit and a SQUID embedded in a two-mode LC resonant circuit. The diabatic states of the qubit correspond to clockwise or counterclockwise persistent currents in the qubit loop [dashed arrow in Fig. 1(a)], with energies controlled by the flux in the loop. The resonator mode of interest is the SQUID plasma mode, depicted by the two solid arrows in Fig. 1(a). The SQUID serves dual purposes: it acts as a tunable coupler between the qubit and the resonator, and it is also used as a sensitive magnetometer for qubit readout [23].

We have investigated two devices with similar layouts but slightly different parameters, both made of aluminum. Device A was designed and fabricated at MIT Lincoln Laboratory and device B was designed and fabricated at NEC. Figure 1(b) shows a spectroscopy measurement of device A versus applied flux, with the qubit flux detuning Φ_{qb} defined as $\Phi_{qb} = \Phi + \Phi_0/2$ and $\Phi_0 = h/2e$. The qubit frequency follows $f_{qb} = \sqrt{\Delta^2 + \varepsilon^2}$, where the tunnel coupling $\Delta = 2.6$ GHz is fixed by fabrication and the energy detuning $\varepsilon = 2I_P\Phi_{qb}/h$ is controlled by the applied flux Φ (I_P is the persistent current in the qubit loop). The resonator frequency is f_r around 2.3 GHz and depends only weakly on Φ_{qb} and I_b. In addition, there are features visible at frequencies corresponding to the sum and difference of the qubit and resonator frequencies, illustrating the coherent coupling between the two systems [3,24].

The system is described by the Jaynes-Cummings Hamiltonian [1,21,25]
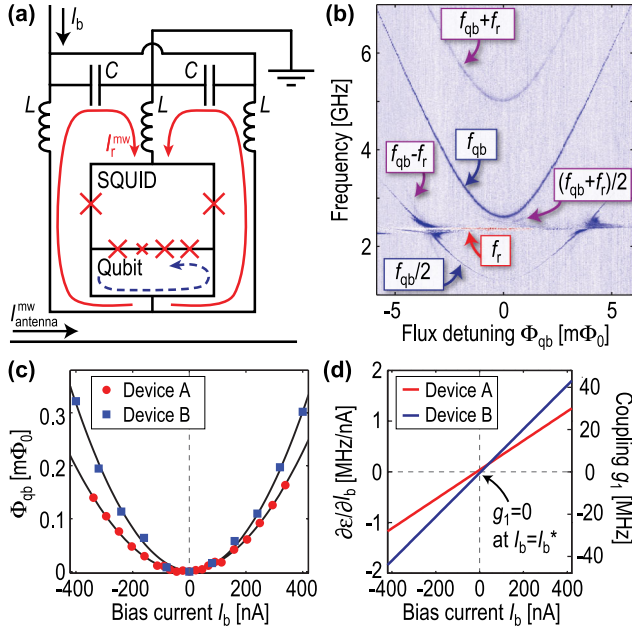
FIG. 1 (color online).   (a) Circuit diagram of the qubit and the
oscillator. The qubit state is encoded in currents circulating
clockwise or counterclockwise in the qubit loop (dashed arrow),
while the mode of the harmonic oscillator is shown by the solid
arrows. (b) Spectrum for device A, showing the qubit and the
harmonic oscillator. In addition, the two-photon qubit $(f_{qb}/2)$
and the qubit $\pm$ resonator $(f_{qb} \pm f_r)$ transitions are visible.
(c) Flux induced in the qubit loop by the dc bias current I_b.
The black lines are parabolic fits. (d) First-order coupling
between the qubit and the ground state of the harmonic oscil-
lator, showing that the coupling is tunable by adjusting I_b. The
coupling is zero at $I_b = I_b^*$, which is slightly offset from $I_b = 0$
due to fabricated junction asymmetry. The derivative $\partial\varepsilon/\partial I_b$ is
calculated from the curves in panel (c). The qubit parameters are
$I_P = 175$ nA for device A and $I_P = 180$ nA for device B. The
resonators have quality factors $Q \approx 100$. The right-hand axis is
calculated using $f_r = 2.2$ GHz and $C_{eff} = 2C = 14$ pF for both
samples.

$$H/h = -\frac{\Delta}{2}\sigma_x - \frac{\varepsilon}{2}\sigma_z + f_r\left(a^\dagger a + \frac{1}{2}\right) + \frac{g_1}{2}(a + a^\dagger)\sigma_z.$$

(1)

Here, a and $a^\dagger$ are the creation/annihilation operators of
the resonator field and g_1 is the dipole coupling between
the qubit and the resonator. In this work, we do not con-
sider higher-order coupling parameters [21,25,26].

The coupling g_1 is mediated by the SQUID. When the
two SQUID junctions are symmetric, the current of the
resonator mode splits equally into the two SQUID arms,
and therefore no net flux is induced into the qubit loop. The
qubit is thus effectively decoupled from the resonator.
However, in the presence of a magnetic field, applying a
dc bias current I_b creates an asymmetric phase drop over
the two SQUID junctions. This causes the resonator cur-
rent to be slightly larger in one of the arms, which will
produce a flux in the qubit loop. The coupling to the
resonator can thus be controlled *in situ* by changing I_b [27].

Figure 1(c) shows the flux induced into the qubit loop as
a function of the dc bias current I_b for the two devices.
Given the similarity in the design, both samples display
similar behavior, with the bias current generating a para-
bolic shift in Φ_{qb} [28,29]. Since Φ_{qb} controls the qubit
energy detuning ε, the first-order qubit-resonator coupling
strength is determined by the derivative $\partial\varepsilon/\partial I_b =
(2I_P/h)(\partial\Phi_{qb}/\partial I_b)$. The bare coupling coefficient between
qubit and resonator is then $g_1 = (\partial\varepsilon/\partial I_b)\delta I_0$, where
$\delta I_0 = \sqrt{2\pi^2 h f_r^3 C_{eff}}$ is the rms amplitude of the vacuum
fluctuations and $C_{eff} = 2C$ is the total capacitance of the
resonant circuit [25]. The derivative $\partial\varepsilon/\partial I_b$ and the cou-
pling g_1 are plotted in Fig. 1(d), determined at dc from the
measured relation between Φ_{qb} and I_b shown in Fig. 1(c).
Note that in both devices g_1 can be tuned over a range of a
few tens of MHz, and that the coupling is turned off at $I_b =
I_b^*$. The device parameters are given in the figure caption.

Having determined the coupling coefficients, we turn to
analyzing how the presence of the resonator influences the
qubit's driven dynamics. Figure 2(a) shows the extracted
Rabi frequency f_{Rabi} of qubit A as a function of I_b, mea-
sured at $f_{qb} = \Delta = 2.6$ GHz. We find that f_{Rabi} changes
by a factor of 5 over the range of the measurement, which
is surprising since both the amplitude and the frequency of
the microwave current $I_{antenna}^{mw}$ in the antenna are kept
constant. The data points were obtained by fitting Rabi
oscillations to decaying sinusoids, a few examples of Rabi
traces for different values of I_b are shown in Fig. 2(b). We
carefully calibrated the flux at each data point, to make
sure that the qubit was driven on resonance and at the
degeneracy point ($\varepsilon = 0$).

To explain the results of Fig. 2(a), we need to consider
the indirect driving of the qubit mediated by the harmonic
oscillator. As seen in Fig. 1(b), at zero flux detuning the
qubit frequency ($f_{qb} = 2.6$ GHz) is relatively close to the
resonator frequency ($f_r = 2.3$ GHz). We therefore expect
the microwave drive in the antenna to off-resonantly in-
duce a microwave current I_r^{mw} in the resonator, which is
proportional to the square root of the average photon
population, $\sqrt{\langle a^\dagger a\rangle}$. By setting $I_b \neq I_b^*$, the coupling be-
tween the resonator and the qubit is turned on, and the
resonator current I_r^{mw} will start driving the qubit. To de-
scribe this indirect driving, we treat the resonator classi-
cally and write the qubit Hamiltonian in Eq. (1) as

$$H_{qb}/h = -1/2[\Delta_{\sigma_x} + [\varepsilon^{dc} + \varepsilon^{mw}\cos(2\pi f t)]\sigma_z].$$ (2)

Here, the drive amplitude ε^{mw} experienced by the qubit
becomes a combination of the drive $\varepsilon_{direct}^{mw}$, due to direct
coupling between antenna and qubit, and the drive
$(\partial\varepsilon/\partial I_b)I_r^{mw}$ mediated by the resonator. We get

$$\varepsilon^{mw} = \sqrt{\left[\varepsilon_{direct}^{mw} + \cos\theta\frac{\partial\varepsilon}{\partial I_b}I_r^{mw}\right]^2 + \left[\sin\theta\frac{\partial\varepsilon}{\partial I_b}I_r^{mw}\right]^2},$$
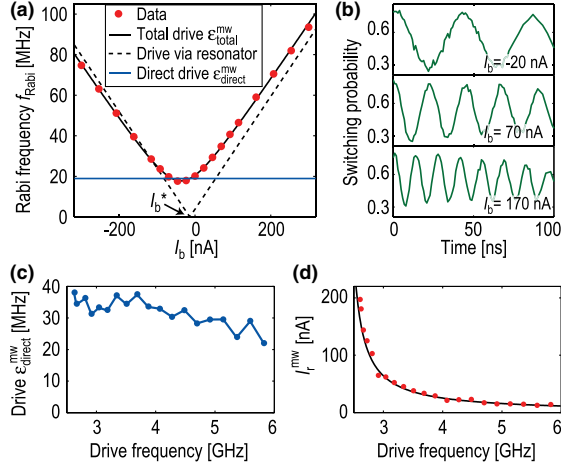
(3)

FIG. 2 (color online).   (a) Rabi frequency of qubit A, measured vs I_b at $\Phi_{qb} = 0$. The driving field seen by the qubit contains two components: one is due to direct coupling to the antenna, the other is due to the coupling mediated by the resonator. (b) Rabi traces for a few of the data points in panel (a). The microwaves in the antenna have the same amplitude and frequency for all traces. (c) Direct coupling between the antenna and the qubit, extracted from measurements similar to the one shown in panel (a). The coupling depends only weakly on frequency. (d) Microwave current in the resonator, induced by a fixed microwave amplitude in the antenna. The black line is a fit to the square root of a Lorentzian, describing the oscillation amplitude of a harmonic oscillator with $f_r = 2.3$ GHz and $Q = 100$.

where $\theta \equiv \theta_d - \theta_r$ is the phase difference between the direct drive and the drive mediated by the resonator. The Rabi frequency due to the drive ε^{mw} depends on the qubit's quantization axis, which changes with the static energy detuning ε_{dc}:

$$f_{Rabi} = \frac{\varepsilon^{mw}}{2} \frac{\Delta}{\sqrt{\varepsilon_{dc}^2 + \Delta^2}}. \tag{4}$$

Fitting the data in Fig. 1(a) to Eqs. (3) and (4) allows us to extract the parameters $\varepsilon_{direct}^{mw}$, I_r^{mw}, and θ. The different drive contributions are plotted together with the data in Fig. 2(a). The direct drive is independent of I_b, while the drive $I_r^{mw}(\partial\varepsilon/\varepsilon I_b)$ mediated by the resonator increases linearly with $|I_b|$, which originates from the linear dependence of g_1 shown in Fig. 1(d). The minimum in Rabi frequency occurs at a value of I_b slightly shifted from the point I_b^* where $g_1 = 0$. This offset appears because of the phase difference θ between the two drive components. The fit gives $\theta = -75°$, which is consistent with a resonator driven above its resonance frequency.

Figures 2(c) and 2(d) show how the two drive components depend on microwave frequency, measured by changing the static flux detuning Φ_{qb} to increase the qubit frequency [see Fig. 1(b)]. The direct drive only depends weakly on frequency (due to cable losses), whereas the drive mediated by the resonator drops sharply as the

qubit-resonator detuning increases. The black curve in Fig. 2(d) is the frequency response of a harmonic oscillator with $f_r = 2.3$ GHz and $Q = 100$, with amplitude normalized to match the data.

To further investigate how the presence of the resonator affects the qubit dynamics at large detunings, we performed measurements on device B. Figure 3(a) shows a spectrum of that device, where the qubit and the resonator mode ($f_r = 2$ GHz) are clearly visible. This device has a larger tunnel coupling ($\Delta = 5.4$ GHz), which allows us to operate the qubit at large frequency detuning from the resonator while still staying at $\varepsilon_{dc} = 0$, where the qubit, to first order, is insensitive to flux noise [28,29]. The qubit-resonator detuning corresponds to several hundred linewidths of the resonator, which is the regime of most interest for quantum information processing [12].

Figure 3(b) shows the Rabi frequency vs bias current I_b of device B, measured at $f_{qb} = \Delta = 5.4$ GHz and for two different values of the microwave drive current $I_{antenna}^{mw}$. Similarly to Fig. 2(a), the Rabi frequency clearly changes with I_b, but the dependence is weaker than in Fig. 2(a) because of the larger frequency detuning. Note that f_{Rabi} scales linearly with $I_{antenna}^{mw}$ for all values of I_b. By fitting the data to Eqs. (3) and (4), we find $\varepsilon_{direct}^{mw}/I_{antenna}^{mw} = 6.4$ MHz/μA, $I_r^{mw}/I_{antenna}^{mw} = 2.4$ nA/μA,   and $\theta = -155°$. The large phase difference θ for device B causes the minimum in f_{Rabi} to shift away from the point at $I_b = I_b^* = 5$ nA where the coupling $g_1 = 0$ [see Fig. 1(d)]. We attribute the large phase shift to influences from a second resonant mode, which is formed by the two L and the two C in the outer loop of Fig. 1(a) [17,30]. For sample B, this mode resonates around 5 GHz.

The results of Figs. 2 and 3 show that the microwave signal mediated by the resonator plays a significant role when driving the qubit, appearing already at moderate qubit-resonator coupling g_1 and persisting even when the two systems are far detuned. The design investigated here
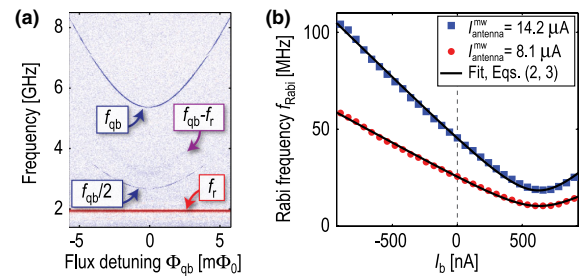

FIG. 3 (color online).   (a) Spectrum of device B. The spectral line at 2 GHz is the resonator, whereas the qubit tunnel coupling is $\Delta = 5.4$ GHz. (b) Rabi frequency vs bias current I_b, measured at $f = 5.4$ GHz and $\Phi_{qb} = 0$ and for two different microwave drive amplitudes $I_{antenna}^{mw}$. Similar to device A, the Rabi frequency depends strongly on I_b, and scales linearly with drive amplitude. The black lines are fits to Eqs. (3) and (4), using the same coupling parameters for both sets of data. Note that the range of I_b in Fig. 3(b) is several times larger than in Fig. 2(a).

allows the coupling to be turned off [$g_1 = 0$ in Fig. 1(d)], but it comes with a drawback: the parameter used to control the coupling (I_b in our setup) also provides a way for low-frequency noise to enter the system. Consider the relation between the f_{Rabi} and I_b in Fig. 3(b): fluctuations δI_b near $I_b = 0$ will cause fluctuations in the amplitude of the drive field seen by the qubit, which will lead to dephasing during driven evolution.

To quantify the dephasing, we linearize the relation between the Rabi frequency and I_b close to $I_b = 0$ as $f = f_0[1 + r\delta I_b]$, where $f_0 = f_{Rabi}(I_b = 0)$ and $r = (\partial f_{Rabi}/\partial I_b)/f_0 = -1.28 \ (\mu A)^{-1}$ is given by Eqs. (3) and (4), or from Fig. 3(b). We model the fluctuations δI_b as normally distributed, with standard deviation σ_I. Assuming the noise to be quasistatic, where the value of δI_b is constant during a single trial but differs from run to run [31,32], we find that the Rabi oscillations decay as

$$\int \frac{e^{-\delta I_b^2/(2\sigma_I^2)}}{\sqrt{2\pi\sigma_I^2}} \cos(2\pi f_0[1 + r\delta I_b]t)d\delta I_b$$

$$= e^{-2(\pi f_0 r\sigma_I t)^2} \cos(2\pi f_0 t). \qquad (5)$$

In addition, the qubit energy relaxation time T_1 gives an exponential contribution to the Rabi decay, with time constant $4T_1/3$ given by the Bloch equations. The Rabi decay also depends on the flux noise at the Rabi frequency, but this contribution can be disregarded when operating the qubit at $\varepsilon_{dc} = 0$, where the qubit is insensitive to first-order flux noise [33]. The total decay envelope $f(t)$ of the Rabi oscillations becomes

$$f(t) = e^{-(3/4T_1)t}e^{-(t/T_\varphi)^2}, \quad \text{with} \quad T_\varphi = 1/(\sqrt{2}\pi f_0 r\sigma_I). \qquad (6)$$

Note that the Gaussian decay constant T_φ due to the effective amplitude fluctuations is inversely proportional to f_0, the average Rabi frequency. This is a consequence of having noise in the coupling between the qubit and the antenna; the effective amplitude fluctuations seen by the qubit will scale with the drive amplitude.

The red circles in Fig. 4(a) show the envelope of Rabi oscillations measured for $f_{Rabi} = 65$ MHz, together with a fit to Eq. (6) The qubit energy relaxation $T_1 = 11.7 \ \mu s$ is known from separate experiments [33], leaving $T_\varphi = 4.3 \ \mu s$ as a fitting parameter. In Fig. 4(b), we plot the Rabi decay time versus f_{Rabi}, extracted from envelopes similar to Fig. 4(a). To capture both the exponential and the Gaussian decay, we plot the time T_e for the envelope to decrease by a factor $1/e$. For the lowest Rabi frequency, the decay time is within 25% of the upper limit set by qubit relaxation, but it decreases with f_{Rabi}, as expected from Eq. (6). The black solid line shows a fit to Eq. (6), giving a value of $\sigma_I = 0.8$ nA for the noise in I_b. The effective fluctuations in the drive amplitude are $r\sigma_I = 0.06\%$. We cannot rule out that part of that noise may be caused by instrument imperfections.
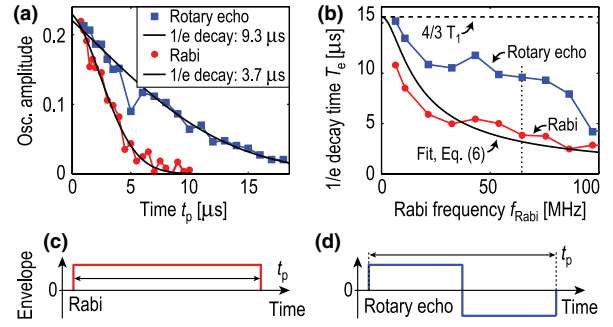

FIG. 4 (color online).   (a) Decay envelopes of the Rabi and rotary-echo sequences for device B, measured with $f_{Rabi} = 65$ MHz and $I_b = 0$. The solid lines are fits to Eq. (6). (b) Decay times for Rabi and rotary echo, extracted from fits similar to the ones shown in panel (a). The dashed line shows the upper limit set by qubit energy relaxation. The dotted line marks the position for the decay envelope shown in panel (a). (c), (d) Schematic diagrams describing the two pulse sequences in (a) and (b). For rotary echo, the phase of the microwaves is rotated by 180° during the second half of the sequence.

Dephasing due to low-frequency fluctuations of the qubit frequency is routinely reduced by performing a Hahn-echo experiment [34]. Similarly, the fluctuations in drive field that cause decay of the Rabi oscillations in Fig. 4(b) can be mitigated with the rotary-echo pulse sequence [22], depicted in Fig. 4(c), which for driven systems is analogous to the Hahn-echo sequence. By shifting the phase of the drive by 180° after a time $t_p/2$, any additional rotations, acquired due to slow fluctuations in the drive amplitude during the first half of the sequence, will cancel out during the reversed rotations in the second half of the sequence.

The blue squares in Fig. 4(a) show the decay of the rotary-echo sequence, measured for $f_{Rabi} = 65$ MHz. The rotary-echo data show a clear improvement compared to the Rabi decay for the same parameters [red circles in Fig. 4(a)]. We fit the rotary-echo data to Eq. (6) and plot the extracted decay times together with the results from Rabi measurements in Fig. 4(b). The rotary-echo signal outperforms the Rabi decay over the full range of Rabi frequencies, and reaches the upper limit set by qubit relaxation ($4T_1/3$) at low frequencies. For intermediate frequencies, the rotary-echo decay times are slightly shorter than $4T_1/3$; we attribute the reduced coherence times to fluctuations in I_b that occur on time scales comparable to the length of the pulse sequence. Noise at frequencies around $1/t_p$ will not be refocused by the reversed drive pulse, since the rotary-echo sequence has similar filtering properties as the Hahn echo [22]. At the highest drive amplitudes ($f_{Rabi} > 100$ MHz), we observe a strong increase in decoherence, probably due to heating. The indirect driving can also be reduced by driving the qubit with two antennas with different amplitudes and phases [35], or by directly applying a phase-shifted microwave signal to the SQUID bias line.

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